# MAK

MATHEMATICS ASSOCIATION OF KENYA

Kenya Mathematical Olympiad

# KMO 10-I

Kenya Mathematical Olympiad 10-Round I

## SENIOR KENYA MATHEMATICS OLYMPIAD

## SCHOOL OF MATHEMATICS, UNIVERSITY OF NAIROBI

**INSTRUCTIONS**

**TIME ALLOWED: 2 HOURS** **JUNE 27, 2019**

1. This question paper consists of **4 printed pages**, including this cover. There are **20 questions**.
2. This is a multiple choice paper with each labeled A, B, C, D, and E. Only **ONE** of these is correct.
3. Each correct answer is **WORTH** 5 marks.
4. For each **INCORRECT** answer, 1 mark will be **DEDUCTED**. There is **NO PENALTY** for unanswered questions.
5. Attempt **ALL** Questions. You **MUST** use a pencil.
6. Rulers, pair of compasses, rough paper and erasers are **ALLOWED**.
7. Calculators, Formula Tables and other Geometrical Instruments are **NOT** permitted.
8. Diagrams are **NOT** necessarily drawn to scale.

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The Committee on the Kenyan Mathematical Olympiads (CKMO) reserves the right to disqualify all scores from a school if it determines that the required security procedures were not followed.



**TURN OVER**

1. Suppose $a, b$ and $c$ are real numbers such that $a+b+c=0$ and $abc=-100$. Let
$$x=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}.$$
Which of the following statements is true?

A. $x>0$ B. $x=0$ C. $-1<x<0$ D. $-100<x<-1$ E. $x<-100$

2. Consider the curves $y=2x^3+6x+1$ and $y=-\frac{3}{x^2}$ in the cartesian plane. Find the number of points at which the two curves intersect.

A. $0$ B. $1$ C. $2$ D. $3$ E. $5$

3. A bag contains $x$ green and $y$ red sweets. A sweet is selected at random from the bag and its colour is noted. It is then replaced into the bag with 10 additional sweets of the same colour. A second sweet is picked randomly next. Find the probability that the second sweet is red.

A. $\frac{y+10}{x+y+10}$ B. $\frac{y}{x+y+10}$ C. $\frac{y}{x+y}$ D. $\frac{x}{x+y}$ E. $\frac{x+y}{x+y+10}$

4. If $a$ and $b$ are integers and $\sqrt{7-4\sqrt{3}}$ is one of the roots of $x^2+ax+b=0$. Find the value of $a+b$.

A. $-3$ B. $-2$ C. $0$ D. $2$ E. $3$

5. Suppose that the three numbers $1$, $a$ and $b$ are three consecutive terms of both an arithmetic progression and a geometric progression. How many possible pairs of $a$ and $b$ are there?

A. $0$ B. $1$ C. $2$ D. $3$ E. $4$

6. What is the value of $\left(\sqrt[6]{8}-18\right)^2$?

A. $4$ B. $6$ C. $8$ D. $10$ E. $12$

7. Let $\alpha$ and $\beta$ be the roots of the quadratic equation $x^2+x-3=0$. Which of the following is the value of $\alpha^3-4\beta^2+20$?

A. $-1$ B. $2$ C. 0 D. $1$ E. $3$

8. Find the smallest positive integer $n$ such that $5^n>1000n$.

A. $4$ B. $5$ C. $6$ D. $7$ E. $8$

9. What is the smallest positive prime factor of $2017^{2019}+2019^{2017}$?

A. $5$ B. $3$ C. $2$ D. $7$ E. $11$

10. What is the largest possible prime that can be written in the form $n^2-12n+27$, where $n$ ranges over all positive integers?

A. $91$ B. $37$ C. $23$ D. $17$ E. $7$

11. Find the sum of all real numbers $x$ satisfying the equation

$$(3^x - 27)^2 + (5^x - 625)^2 = (3^x + 5^x - 652)^2$$

A. $\mathbf{7}$ B. $\mathbf{8}$ C. $\mathbf{3}$ D. $\mathbf{4}$ E. $\mathbf{11}$

12. Let $a, b$ and $c$ be real numbers such that

$$a = 8 - b \text{ and } c^2 = ab - 16.$$

Find the value of $a + c$.

A. $\mathbf{4}$ B. $\mathbf{8}$ C. $\mathbf{0}$ D. $\mathbf{16}$ E. $\mathbf{24}$

13. In a triangle $\Delta ABC$, suppose $AC = 2BC \sin B$. Find the angle at the vertex $A$ if it is an acute angle.

A. $\mathbf{30^\circ}$ B. $\mathbf{120^\circ}$ C. $\mathbf{60^\circ}$ D. $\mathbf{90^\circ}$ E. $\mathbf{45^\circ}$

14. In the figure below $ABCD$ is a square and

$$\frac{AE}{EB} = \frac{BF}{FC} = \frac{CG}{GD} = \frac{DH}{HA} = \frac{m}{n}$$

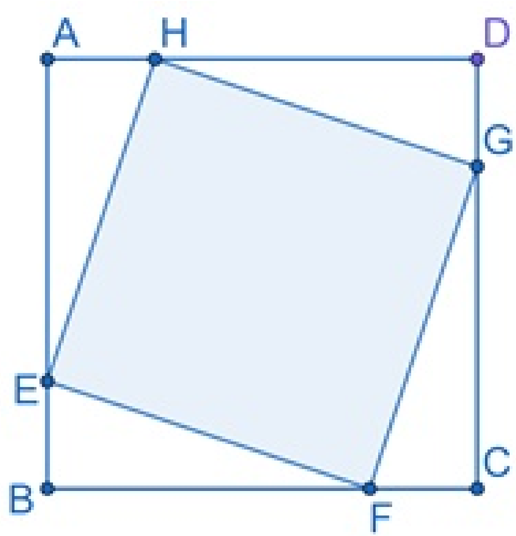


Figure 1: Figure for Problem 14

The ratio of the area of $EFGH$ to the area of $ABCD$ is

A. $m^2 + n^2$ B. $\left(\frac{m}{n}\right)^2$ C. $\left(\frac{n}{m}\right)^2$ D. $\frac{m^2 + n^2}{(m+n)^2}$ E. $\frac{mn}{1}$

**TURN OVER**

15. If $\sin x = 3\cos x,$ find the value of $900 \sin x \cos x.$

A. **270** B. **900** C. **100** D. **300** E. **450**

16. The integer part of the fraction

$$\frac{1}{\frac{1}{1984} + \frac{1}{1985} + \cdots + \frac{1}{1999}}$$

is

A. **123** B. **122** C. **124** D. **126** E. **125**

17. Find the total number of triangles such that the length (in cm) of all three sides of each triangle are positive integers and the length of the longest side of each triangle is $15cm$ and one of the shorter sides does not exceed **9**

A.**31** B. **32** C. **43** D. **44** E. **15**

18. Let $a$ and $b$ be positive numbers such that

$$\frac{1}{a} - \frac{1}{b} - \frac{1}{a+b} = 0.$$

Find the value of $\left(\frac{b}{a} + \frac{a}{b}\right)^2.$

A. **4** B. **5** C. **6** D. **7** E. **8**

19. Let $x, y$ and $z$ be nonnegative numbers. Suppose that $x + y = 10, y + z = 8$ and $S = x + z.$ What is the sum of the maximum and the minimum value of $S?$ (nonnegative numbers are those greater than or equal to zero)

A. **16** B. **60** C. **20** D. **24** E. **26**

20. Let $x$ be a number such that $x + \frac{1}{x} = 4.$ Find the value of $x^3 + \frac{1}{x^3}.$

A. **48** B. **50** C. **52** D. **54** E. **64**

# END