SCHOOL OF MATHEMATICS

UNIVERSITY OF NAIROBI

KENYA MATHEMATICS OLYMPIAD

KMO 2016 - FIRST ROUND

Time allowed: 2 hours

SEPTEMBER 20, 2016

INSTRUCTIONS

1. This question paper consists of **4 printed pages**, including this cover.
2. Please read the **INSTRUCTIONS** for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret questions, so we do not answer questions about exam problems during the exam.
3. This is a multiple choice paper with a total of **20 QUESTIONS**.
4. Each question is followed by answers labeled A, B, C, D, and E. Only **ONE** of these is correct.
5. Attempt **ALL** Questions. You **MUST** use a pencil.
6. Each correct answer is **WORTH** 5 marks.
7. For each **INCORRECT** answer, 1 mark will be **DEDUCTED**. There is **NO PENALTY** for unanswered questions.
8. Rulers, pair of compasses, rough paper and erasers are **ALLOWED**.
9. Calculators, Formula tables and other Geometrical Instruments are **NOT** permitted.
10. Diagrams are **NOT** necessarily drawn to scale.
11. **Answers** and **Solutions** will be available at **www.mathskenya.org**

TURN OVER

PROBLEMS

1. Determine the value of $\dfrac{17917}{(-17917)^2+(-17916)(17918)}$.

   (A). 17915 (B). 17916 (C). 17000 (D). 17918 (E). 17917

2. Given $x-y=12$. Find the value of x^3-y^3-36xy.

   (A). 1327 (B). 1728 (C). 1729 (D). 1730 (E). 1731

3. Consider the sequence of integers $\{1,2,3,4,\ldots\}$. A new sequence is obtained by deleting all the perfect squares. The 2016th term of the new sequence is

   (A). 2061 (B). 2062 (C). 2063 (D). 2064 (E). 2060

4. Suppose $-2<x<2$ and $-2<y<2$ and $xy=1$. What is the minimum value of $u=\dfrac{4}{4-x^2}+\dfrac{9}{9-y^2}$.

   (A). $\dfrac{12}{7}$ (B). $\dfrac{24}{11}$ (C). $\dfrac{12}{5}$ (D). $\dfrac{9}{4}$ (E). $\dfrac{8}{5}$

5. Let a,b,c,d be positive integers such that $\log_a b=\dfrac{3}{2}$ and $\log_c d=\dfrac{5}{4}$. If $a-c=9$, then the value of $b-d$ is

   (A). 9 (B). 35 (C) 93. (D). 24 (E) 83

6. The smallest integer greater than or equal to $\left(2+\sqrt{3}\right)^2$ is:

   (A). 6 (B). 13 (C) 14. (D). 16 (E) 11

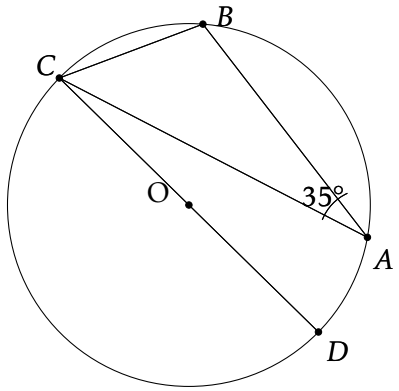
7. Let k be a real number such that the inequality $\sqrt{x-4}+\sqrt{8-x}\geq k$ has a solution. The maximum value of k is

   (A). 4 (B). $\sqrt{2}$ (C) 2. (D). $2\sqrt{2}$ (E) 8

8. When a 3 digit number x is divided by 2,3,5, 6 and 7, the remainders are all 1. Find the smallest such 3 digit number?

   (A). 421 (B). 210 (C) 1261. (D). 211 (E) 127

9. In the diagram O is the center of the circle and angle $\angle BAC = 35^0$.



What is the size of angle $\angle BCD$?

(A). 45 (B). 65 (C). 35 (D). 40 (E). 55

10. Given $x + y = \sqrt{3\sqrt{5} - \sqrt{2}}$ and $x - y = \sqrt{3\sqrt{2} - \sqrt{5}}$, find the value of xy.

(A). $\sqrt{8}$ (B). $\sqrt{10} - 2$ (C). $2\sqrt{3}$ (D). $\sqrt{2} - \sqrt{3}$ (E). $\sqrt{5} - \sqrt{2}$

11. The numbers 3,4,7 and 8 are used to form a 4 digit number without repeating any digit. How many such 4 digit numbers are divisible by 11?

(A). 4 (B).8 (C). 6 (D). 7 (E). 10

12. Given that $x = 2a^5 = 3b^2$ where a and b are positive integers. Find the least possible value of b.

(A). 36 (B). 15 (C). 42 (D). 72 (E). 18

13. Suppose x_1, x_2, x_3 are the roots of $(13 - x)^3 + (17 - x)^3 = (30 - 2x)^3$. What is the sum of $x_1 + x_2 + x_3$?

(A). 30 (B). 45 (C). 37 (D). 33 (E). 60

14. An arithmetic sequence with 10 terms has common difference $d > 0$. If the absolute value of each term is a prime number, find the smallest possible value of d?

(A). 25 (B). 20 (C). 30 (D). 15 (E). 45

15. Suppose $\frac{1}{x} = \frac{3}{y+z} = \frac{4}{z+x}$. What is the value of $\frac{y-x}{z}$?

(A). $\frac{1}{4}$ (B). $\frac{1}{3}$ (C). $-\frac{1}{3}$ (D). 3 (E). 0

16. How many zeros does the product of $75^5, 150^4$ and 2016^3 end with?

(A). 18 (B). 15 (C). 13 (D). 14 (E). 21

17. Given that $\sqrt{x+2y} + \sqrt{x^2-16} = 0$. Which of the following is a possible value for $y-x$?

(A). 4 (B). −4 (C). 6 (D). 8 (E). 14

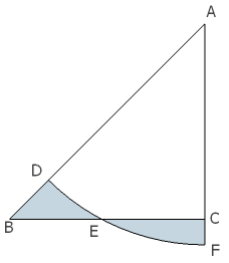
18. There are 12 lamps, initially all off, each of which comes with a switch. When a switch is pressed, a lamp which is off will be turned on, and a lamp which is on will be turned off. Now one is allowed to press exactly 5 different switches in each round. What is the minimum number of rounds needed so that all lamps will be turned turned on?

(A). 6 (B). 3 (C). 2 (D). 8 (E). 4

19. Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x. Find the last seven digits (from left to right) of the number $\left\lfloor \frac{10^{2016}}{10^{672}+2015} \right\rfloor$.

(A). 4060224 (B). 4060215 (C). 4060216 (D). 4062015 (E). 4062016

20. In the following right angled triangle ABC, $AC = 1$, $BC = 1$ and DEF is an arc of a circle with center A.



Suppose the shaded areas BDE and CEF are equal and $AD = \frac{x}{\sqrt{\pi}}$. What is the value of x^2?

(A). 4 (B). 2 (C). 8 (D). 16 (E). 25

END